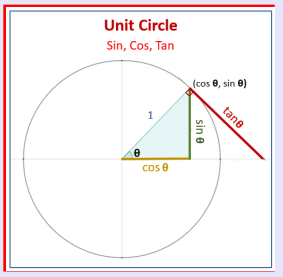
Trigonometry

Lecture 46

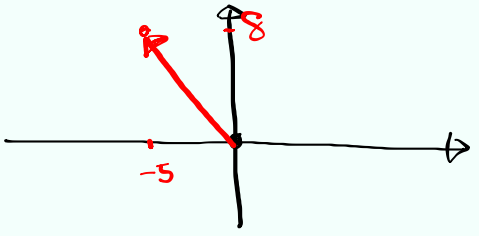


Feb 19-8:47 AM

Given $P(3,2)$, $Q(-2,10)$

1) Find $\overrightarrow{PQ}$ $\quad \overrightarrow{PQ} = \langle -2-3, 10-2 \rangle = \langle -5, 8 \rangle$

2) Draw $\overrightarrow{PQ}$ in Standard Position



3) Find $|\overrightarrow{PQ}|$

$= \sqrt{(-5)^2 + (8)^2} = \sqrt{25+64} = \sqrt{89}$

Nov 21-10:29 AM

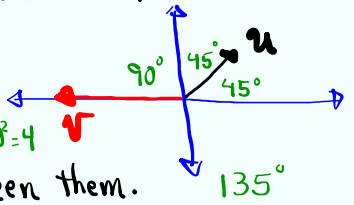
$u = \langle 2, 2 \rangle$ $\quad v = \langle -4, 0 \rangle = -4\langle 1, 0 \rangle = -4i$

1) write u & v using unit vector i & j.

$u = \langle 2, 2 \rangle$

$= \langle 2, 0 \rangle + \langle 0, 2 \rangle = 2\langle 1, 0 \rangle + 2\langle 0, 1 \rangle = 2i + 2j$

2) Draw u & v in standard position.



$u \cdot v = 2(-4) + 2(0) = -8$

$|u| = \sqrt{2^2 + 2^2} = \sqrt{8}$ $\quad |v| = \sqrt{(-4)^2 + 0^2} = 4$

3) Find the angle between them. $135°$

$\cos\theta = \dfrac{u \cdot v}{|u||v|}$ $\quad \cos\theta = \dfrac{-8}{\sqrt{8} \cdot 4} = \dfrac{-8}{2\sqrt{2} \cdot 4}$

$= \dfrac{-1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$

$\cos^{-1}\left(-\dfrac{\sqrt{2}}{2}\right) = 135°$

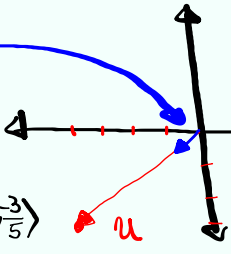
Nov 21-10:33 AM

Find a unit vector in the direction of $u = \langle -4, -3 \rangle$. Draw u & the unit vector.

unit vector $\dfrac{1}{|u|} u$

$= \dfrac{1}{5}\langle -4, -3 \rangle = \left\langle \dfrac{-4}{5}, \dfrac{-3}{5} \right\rangle$

$= \langle -.8, -.6 \rangle$



$|u| = \sqrt{(-4)^2 + (-3)^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= \boxed{5}$

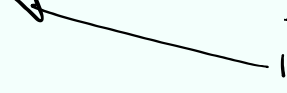
$|\langle -.8, -.6 \rangle| = \sqrt{(-.8)^2 + (-.6)^2}$

$= \sqrt{.64 + .36}$

$= \sqrt{1}$

$= 1$

unit vector



Nov 21-10:42 AM

$u = \langle 6, 8 \rangle$ $\quad v = \langle 5, 0 \rangle$

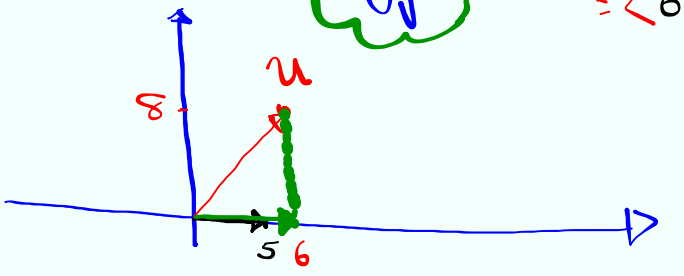
$u \cdot v = 6(5) + 8(0) = 30$

$v \cdot v = 5 \cdot 5 + 0 \cdot 0 = 25$

1) Find $\text{Proj}_v u$

$$\text{Proj}_v u = \frac{u \cdot v}{v \cdot v} v = \frac{30}{25} \langle 5, 0 \rangle = \frac{6}{5} \langle 5, 0 \rangle = \left\langle \frac{6}{5} \cdot 5, \frac{6}{5} \cdot 0 \right\rangle = \langle 6, 0 \rangle$$

2) Draw u, v, and $\text{Proj}_v u$

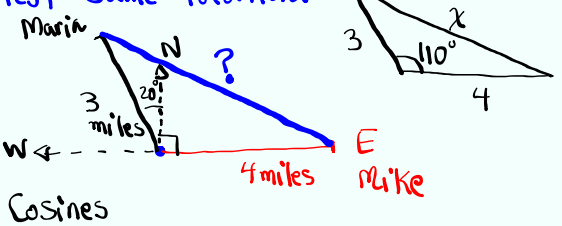


Nov 21-10:47 AM

Mike walks 4 mph in east direction.

At the same time, Maria walks 3 mph with bearing of N 20° W.

How far apart are they after 1 hr if they left same location?



Law of Cosines

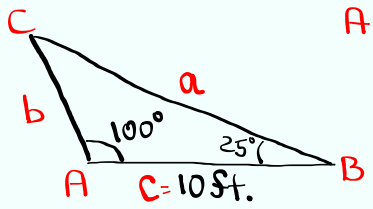
$x^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cdot \cos 110°$

$x^2 \approx 33.208 \qquad x \approx 5.763$

about 6 miles apart.

Nov 21-10:53 AM

Solve the triangle below



$A + B + C = 180°$

$\boxed{C = 55°}$

Law of Sines $\quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$$\frac{\sin 100°}{a} = \frac{\sin 25°}{b} = \frac{\sin 55°}{10}$$

$$\frac{\sin 100°}{a} = \frac{\sin 55°}{10}$$

$$a \sin 55° = 10 \sin 100°$$

$$a = \frac{10 \sin 100°}{\sin 55°}$$

$\boxed{a \approx 12 \text{ ft}}$

do a similar work to find b.

Nov 21-11:00 AM

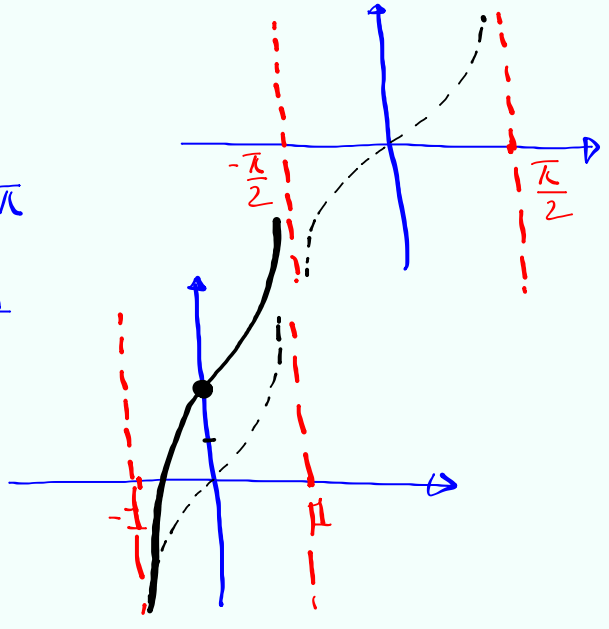
Graph $y = 2 + \tan\left(\frac{\pi x}{2}\right)$

$y = \tan x$

$$-\frac{\pi}{2} < \frac{\pi x}{2} < \frac{\pi}{2}$$

$$-\pi < \pi x < \pi$$

$$-1 < x < 1$$



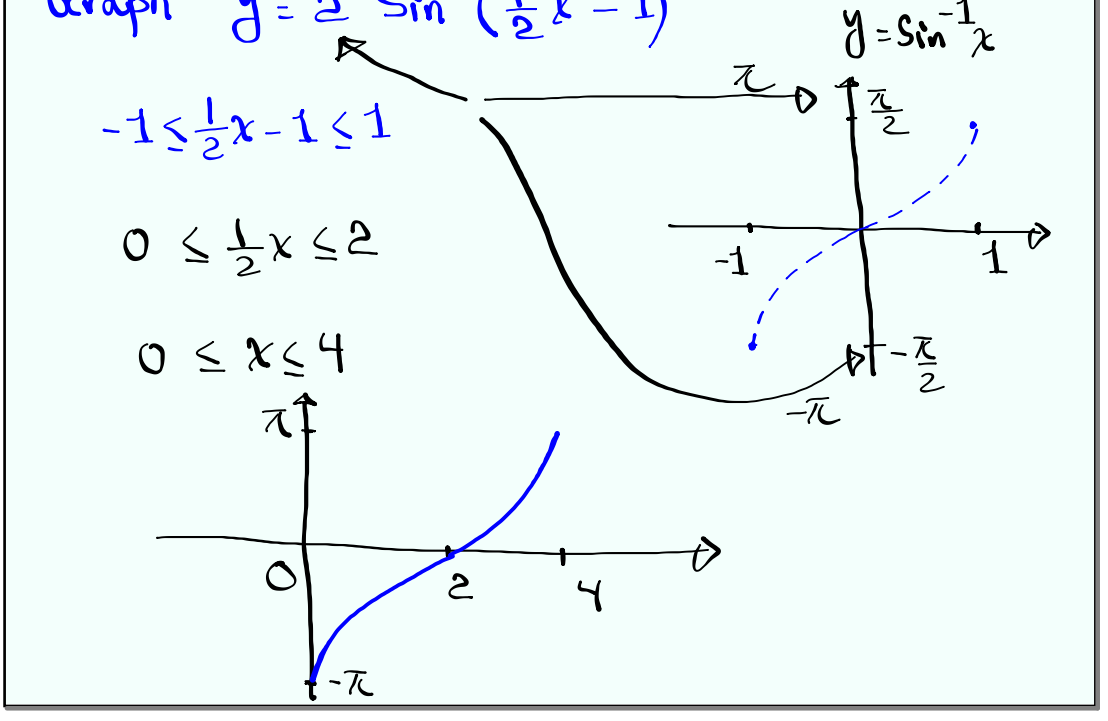
Nov 21-11:04 AM

Graph $y = 2\sin^{-1}\left(\frac{1}{2}x - 1\right)$

$-1 \le \frac{1}{2}x - 1 \le 1$

$0 \le \frac{1}{2}x \le 2$

$0 \le x \le 4$



Nov 21-11:09 AM

Solve $\tan^2 2x - 3 = 0$

$\tan^2 2x = 3$

$\tan 2x = \pm\sqrt{3}$

$\tan 2x = \sqrt{3}$

QI, QIII

RA $\Rightarrow 60°$

$2x = 60° + K\cdot 180°$

$x = 30° + K\cdot 90°$

$\tan 2x = -\sqrt{3}$

QII, QIV

RA $\Rightarrow 60°$

$2x = 180° - 60° + K\cdot 180°$

$x = 60° + K\cdot 90°$

$2x = 360° - 60° + K\cdot 180°$

$x = 150° + K\cdot 90°$

$2x = 180° + 60° + K\cdot 180°$

$x = 120° + K\cdot 90°$

K=0 30°, 60°, 120°, 150°

K=1 ~~120°~~, ~~150°~~, 210°, 240°

K=2 ~~210°~~, ~~240°~~, 300°, 330°

Nov 21-11:14 AM

Solve $\sin x + \cos x = \sqrt{2}$ on $[0, 2\pi)$

Square both sides

$(\sin x + \cos x)^2 = (\sqrt{2})^2$

$\sin^2 x + 2\sin x \cos x + \cos^2 x = 2$

$1 + 2\sin x \cos x = 2$

$\sin 2x = 1$

RA $\frac{\pi}{2}$

$2x = \frac{\pi}{2} + K \cdot 2\pi$

$x = \frac{\pi}{4} + K \cdot \pi$

$K=0$ $\frac{\pi}{4}$

$K=1$ $\frac{\pi}{4} + \pi = \frac{5\pi}{4}$

$K=2$ $\frac{\pi}{4} + 2\pi$

$\frac{\pi}{4}$ $\frac{5\pi}{4}$

$\sin \frac{\pi}{4} + \cos \frac{\pi}{4} =$

$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\sin \frac{5\pi}{4} + \cos \frac{5\pi}{4} =$

$-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$

Nov 21-11:22 AM

$\sin x + \cos x = \sqrt{2}$

Divide by $\sqrt{2}$

$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{\sqrt{2}}{\sqrt{2}}$

$\sin x \cdot \frac{\sqrt{2}}{2} + \cos x \cdot \frac{\sqrt{2}}{2} = 1$

$\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4} = 1$

$\sin\left(x + \frac{\pi}{4}\right) = 1$

$x + \frac{\pi}{4} = \frac{\pi}{2} + K \cdot 2\pi$

$x = \frac{\pi}{2} - \frac{\pi}{4} + K \cdot 2\pi$

$x = \frac{\pi}{4} + K \cdot 2\pi$

$K=0$ $\frac{\pi}{4}$

$K=1$

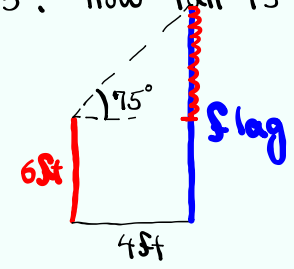
$\frac{\pi}{4} + 2\pi$

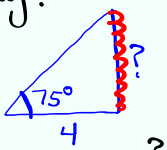
Nov 21-11:30 AM

Jose is 4 ft from a flag.

Jose is 6 ft tall.

His angle of elevation to the top of flag is 75°. How tall is the flag?





$\tan 75^\circ = \frac{?}{4}$

$? = 4 \tan 75^\circ$

≈ 15 ft

Flag is 21 ft tall

Nov 21-11:35 AM